ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt{1+x}-\sqrt{1-x}+\sqrt{1-x^2}=1$$

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Denote:

$$\begin{cases}\sqrt{1+x}=a\\ \sqrt{1-x}=b\end{cases}\Rightarrow a^2+b^2=2$$

$$\sqrt{1+x}-\sqrt{1-x}+\sqrt{1-x^2}=1\Rightarrow a-b+ab=1$$

$$a(1+b)-(1+b)=0\Rightarrow(1+b)(a-1)=0$$

If $b+1=0\Rightarrow b>-1$. But $6>0$. Contradiction!

$$a-1=0\Rightarrow a=1\Rightarrow\sqrt{1+x}=1\Rightarrow x=0$$