

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt{x} + \sqrt{1-x} = 1 + \sqrt{x(1-x)}$$

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$$x \geq 0$$

$$1-x \geq 0 \Rightarrow x \in [0, 1]$$

Denote:

$$\begin{cases} \sqrt{x} = a \\ \sqrt{1-x} = b \end{cases} \Rightarrow a^2 + b^2 = 1$$

$$\sqrt{x} + \sqrt{1-x} = 1 + \sqrt{x(1-x)} \Rightarrow a + b = 1 + ab$$

$$\begin{aligned} ab - a + 1 - b &= 0 \\ (b-1)(a-1) &= 0 \Rightarrow a(b-1) - (b-1) = 0 \end{aligned}$$

$$a-1=0 \Rightarrow a=1 \Rightarrow 1^2 + b^2 = 1 \Rightarrow b=0 \Rightarrow \begin{cases} \sqrt{x} = 1 \\ \sqrt{1-x} = 0 \end{cases} \Rightarrow x=1$$

$$b-1=0 \Rightarrow b=1 \Rightarrow a^2 + 1^2 = 1 \Rightarrow a=0 \Rightarrow \begin{cases} \sqrt{x} = 0 \\ \sqrt{1-x} = 1 \end{cases} \Rightarrow x=0 \Rightarrow S \in \{0, 1\}.$$