

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$(x+1)(x+3)(x+5)(x+7) = 9$$

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$$(x+1)(x+3)(x+5)(x+7) = 9, \quad (x+1)(x+7)(x+3)(x+5) = 9$$

$$(x^2 + 8x + 7)(x^2 + 8x + 15) = 9$$

Denote: $x^2 + 8x = y \Rightarrow (y+7)(y+15) = 9$

$$y^2 + 22y + 105 - 9 = 0 \Rightarrow y^2 + 22y + 96 = 0$$

$$\Delta = 484 - 384 = 100$$

$$y_1 = \frac{-22 - 10}{2} = -\frac{32}{2} = -16; y_2 = \frac{-22 + 10}{2} = \frac{-12}{2} = -6$$

$$x^2 + 8x = -16 \Rightarrow x^2 + 8x + 16 = 0 \Rightarrow (x+4)^2 = 0 \Rightarrow x_1 = x_2 = -4$$

$$x^2 + 8x = -6 \Rightarrow x^2 + 8x + 8 = 0$$

$$\Delta = 64 - 32 = 32, \quad x_{3,4} = \frac{-8 \pm 4\sqrt{2}}{2} = -4 \pm 2\sqrt{2}$$