

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt[3]{(1+x)^2} + \sqrt[3]{(1-x)^2} - \sqrt[3]{1-x^2} = 1$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

Denote: $a = \sqrt[3]{1+x}$; $b = \sqrt[3]{1-x}$

$$a^3 + b^3 = 1 + x + 1 - x = 2 \Rightarrow a^3 + b^3 = 2, \quad a^2 + b^2 - ab = 1$$

$$(a+b)(a^2 + b^2 - ab) = 2 \Rightarrow a + b = 2$$

$$a^2 + b^2 - ab = 1 \Rightarrow S^2 - 2P - P = 1$$

Where: $S = a + b$; $P = ab$, $4 - 3P = 1 \Rightarrow 3P = 3 \Rightarrow P = 1$; $S = 2$

$$\left. \begin{array}{l} S = a + b = 2 \\ P = ab = 1 \end{array} \right\} \Rightarrow a = b = 1$$

$$\sqrt[3]{1+x} = 1 \Rightarrow 1+x = 1 \Rightarrow x = 0$$

$$\sqrt[3]{1-x} = 1 \Rightarrow 1-x = 1 \Rightarrow x = 0$$