

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt[3]{1+x} + \sqrt[3]{1-x} = \sqrt[3]{1-x^2} + 1$$

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$$\left. \begin{array}{l} \sqrt[3]{1+x} = a \Rightarrow 1+x = a^3 \\ \sqrt[3]{1-x} = b \Rightarrow 1-x = b^3 \end{array} \right\} \Rightarrow a^3 + b^3 = 2$$

Denote: $S = a + b$; $P = ab$

$$\sqrt[3]{1+x} + \sqrt[3]{1-x} = \sqrt[3]{(1-x)(1+x)} + 1$$

$$a + b = ab + 1 \Rightarrow S = P + 1$$

$$a^3 + b^3 = 2 \Rightarrow S^3 - 3SP = 2 \Rightarrow (P+1)^3 - 3P(P+1) = 2$$

$$P^3 + 3P^2 + 3P + 1 - 3P^2 - 3P = 2$$

$$P^3 + 1 = 2 \Rightarrow P^3 = 1 \Rightarrow P = 1$$

$$S = 2; P = 1 \Rightarrow a = b = 1$$

$$\sqrt[3]{1+x} = 1 \Rightarrow 1+x = 1 \Rightarrow x = 0$$