

ROMANIAN MATHEMATICAL MAGAZINE

Solve for reals:

$$x^2 - x - 1 = \sqrt{2x + 1}$$

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$$\text{Domain } 2x + 1 \geq 0 \text{ or } x \geq -\frac{1}{2}$$

$$x^2 - x - 1 = \sqrt{2x + 1} \text{ or } (x^2 - x - 1)^2 = 2x + 1$$

$$\text{or } x^4 + x^2 + 1 - 2x^3 - 2x^2 + 2x = 2x + 1$$

$$\text{or } x^2(x^2 - 2x - 1) = 0 \Rightarrow x = 0 \text{ or } x^2 - 2x - 1 = 0 \Rightarrow x = 1 \pm \sqrt{2}.$$

Now for $x = 0$ given equation not satisfied

$$\text{for } x = 1 - \sqrt{2}, L.H.S = (1 - \sqrt{2})^2 - (1 - \sqrt{2}) - 1 = 1 - \sqrt{2} < 0$$

$$R.H.S = \sqrt{2(1 - \sqrt{2}) + 1} > 0 \text{ so not satisfied}$$

so required solution $x = 1 + \sqrt{2}$.