

ROMANIAN MATHEMATICAL MAGAZINE

Solve for reals:

$$x^3 - x + 1 = \sqrt[3]{2x - 1}$$

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$$\text{Let } y = \sqrt[3]{2x - 1} \text{ then } y^3 = 2x - 1 \text{ (1)}$$

$$\text{From given equation we have } x^3 - x + 1 = y \text{ (2)}$$

Let we put $y = x$ in equation (1) then we get $x^3 - x + 1 = x$ or

$$x^3 - 2x + 1 = 0 \text{ or } (x - 1)(x^2 + x - 1) = 0 \Rightarrow$$

$$x = 1 \text{ and } x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$$

Now if we put $y = x$ in equation (2) we get $x^3 = 2x - 1$ or $x^3 - 2x + 1 = 0$ which is same as previous equation, so required solution:

$$x = 1, \frac{-1 \pm \sqrt{5}}{2}$$