ROMANIAN MATHEMATICAL MAGAZINE

$$\textit{Find the conditions on a and b so that following equation has 3 distinct solutions}$$

$$x^4-2(a^2+b^2-1)x^2+(a^2-b^2+1)^2-4a^2=0$$

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$$x^4-2(a^2+b^2-1)x^2+(a^2-b^2+1)^2-4a^2=0\ \ (1)$$

$$Let\ y=x^2\ then\ from(1)we\ get\ y^2-2(a^2+b^2-1)y+(a^2-b^2+1)^2-4a^2=0\ (2)$$

$$since\ y=x^2\ if\ both\ roots\ y_1,y_2>0\ then\ eqution\ (1)has\ 4\ distinct\ root.$$

$$If\ one\ root\ is\ 0\ then\ \ y=0\ and\ other\ positive\ root$$

$$y_1>0\ and\ equation\ (1)has\ 3\ distinct\ roots\ as\ 0,\sqrt{y_1},-\sqrt{y_1}$$

$$We\ put\ x=0\ in\ (2)we\ get$$

$$(a^2-b^2+1)^2-4a^2=0\ or,(a^2-b^2+1+2a)(a^2-b^2+1-2a)=0$$

$$or\ (a^2-b^2+1+2a)=0\Rightarrow(a+1)^2=b^2\ or,a=b-1$$

$$(a^2-b^2+1-2a)=0\ \Rightarrow(a-1)^2=b^2\ or\ \ b=a-1$$

$$From(2)sum\ of\ the\ two\ roots$$

$$2(a^2+b^2-1)>0\ (as\ roots\ are\ \ 0,y_1>0)\ or\ a^2+b^2>1$$

$$So\ required\ conditions:a=b-1\ ,b=a-1\ ,a^2+b^2>1$$