ROMANIAN MATHEMATICAL MAGAZINE

***Prove or disprove that:***

***The roots of the equation $x^3+ax^2+bx+c=0$ are in A.P.***

***if $4a^3=9ab+27c$***

*Proposed by Neculai Stanciu-Romania*

*Solution by Tapas Das-India*

$$\text{Let } p-d, p, p+d \text{ are roots of the equation } x^3+ax^2+bx+c=0$$

$$\text{then } p-d+p+p+d=-a \text{ or}, d=-\frac{a}{3}\ (1)$$

$$(p-d)p(p+d)=-c \text{ or}, p(p^2-d^2)=-c\ (2)$$

$$(p-d)p+p(p+d)+(p-d)(p+d)=b \text{ or}, 3p^2-d^2=b\ (3)$$

$$\text{from } (1)\&(3) \text{ we get } d^2=3\left(-\frac{a}{3}\right)^2-b=\frac{a^2}{3}-b\ (4)$$

$$\text{From } (2)\&(4) \text{ we get}, -\frac{a}{3}\left(\frac{a^2}{9}-\left(\frac{a^2}{3}-b\right)\right)=-c \text{ or}, -\frac{a}{3}\left(b-\frac{2a^2}{9}\right)=-c$$

$$\text{or } 2a^3=9ab-27c\ , \text{so the given condition is not true.}$$