ROMANIAN MATHEMATICAL MAGAZINE

If $a > 1, \lambda > 1$ fixed. Solve for reals:

$$x + \sqrt{\lambda^x \log_a(x + 2a)} = \sqrt{\log_a(x + 2a)}$$

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Solution by Tapas Das-India

$$Let\ y = \sqrt{\log_a(x + 2a)} \geq 0\ then:$$
$$x + \sqrt{\lambda^x \log_a(x + 2a)} = \sqrt{\log_a(x + 2a)}$$
$$\Rightarrow x + y\lambda^{\frac{x}{2}} = y\ \Rightarrow x = y\left(1 - \lambda^{\frac{x}{2}}\right)\ (1)$$
$$Case\ 1.\ If\ x > 0\ then\ \lambda^{\frac{x}{2}} > 1\ as\ \lambda > 1\ so\ R.H.S\ part\ of\ (1) < 0$$
$$but\ L.H.S > 0\ contradiction.$$
$$Case\ 2.\ If\ x < 0\ then\ \lambda^{\frac{x}{2}} < 1\ as\ \lambda > 1\ so\ R.H.S\ part\ of\ (1) \geq 0$$
$$but\ L.H.S < 0\ contradiction.$$
$$Clearly, x = 0\ satisfy\ equation\ (1)$$
$$so, required\ solution\ x = 0$$