ROMANIAN MATHEMATICAL MAGAZINE

If $\lambda \in R$, $n \geq 0$ fixed then solve for reals:

$$x^2 + 2(x-\lambda)\sqrt{x-\lambda} + (1-2\lambda)x + \lambda(\lambda-1) = n^2(n+1)^2$$

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Solution by Tapas Das-India

$$Let\ t = \sqrt{x-\lambda}\ , t \geq 0\ so, x = t^2 + \lambda$$
$$Domain:\ x \geq \lambda$$

$$x^2 + 2(x-\lambda)\sqrt{x-\lambda} + (1-2\lambda)x + \lambda(\lambda-1) =$$
$$= (t^2+\lambda)^2 + 2t^2.t + (1-2\lambda)(t^2+\lambda) + \lambda(\lambda-1) =$$
$$= t^4 + 2t^3 + t^2 = t^2(t+1)^2$$

$$x^2 + 2(x-\lambda)\sqrt{x-\lambda} + (1-2\lambda)x + \lambda(\lambda-1) = n^2(n+1)^2$$

$$t^2(t+1)^2 = n^2(n+1)^2\ or\ t(t+1) = n(n+1)(as\ n, t > 0)\ or\ t = n$$

$$t = \sqrt{x-\lambda} = n\ or\ x = n^2 + \lambda$$