ROMANIAN MATHEMATICAL MAGAZINE

For $a > 1$fixed then solve for real numbers:

$$\left(\frac{1}{a}\right)^{x+1} + \left(\frac{1}{a(a+1)}\right)^{x} - \sqrt{a}\left(\frac{\sqrt{a}}{a(a+1)}\right)^{x} = 1$$

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$$\text{Let } s = a^{\frac{x+1}{2}} > 0\, , y = \left(a(a+1)\right)^{-x} > 0$$

$$\left(\frac{1}{a}\right)^{x+1} = a^{-(x+1)} = s^{-2}\, , \left(\frac{1}{a(a+1)}\right)^{x} = y,$$

$$\sqrt{a}\left(\frac{\sqrt{a}}{a(a+1)}\right)^{x} = a^{\frac{1}{2}}.a^{\frac{x}{2}}.\left(a(a+1)\right)^{-x} = a^{\frac{x+1}{2}}.\left(a(a+1)\right)^{-x} = sy$$

$$\left(\frac{1}{a}\right)^{x+1} + \left(\frac{1}{a(a+1)}\right)^{x} - \sqrt{a}\left(\frac{\sqrt{a}}{a(a+1)}\right)^{x} = 1$$

$$s^{-2} + y - sy = 1 \text{ or } 1 + ys^2 - s^3y = s^2$$

$$1 - s^2 + ys^2(1-s) = 0 \ \text{ or} (1-s)(1+s+s^2y) = 0$$

$$as\ s = a^{\frac{x+1}{2}} > 0 \text{ since } a > 1, y = \left(a(a+1)\right)^{-x} > 0 \text{ so } (1+s+s^2y) \text{ can not be } 0$$

$$so\ 1 - s = 0 \text{ or } s = 1 \Rightarrow a^{\frac{x+1}{2}} = 1 \Rightarrow \frac{x+1}{2} = 0 \text{ or } x = -1.$$