ROMANIAN MATHEMATICAL MAGAZINE

If $\lambda \in R$ then solve for reals:

$$\sqrt[3]{x-\lambda+1}+\sqrt[3]{x-\lambda}+\sqrt[3]{x-\lambda-1}=0$$

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Solution by Tapas Das-India

$$Let\ y=x-\lambda\ then \sqrt[3]{x-\lambda+1}+\sqrt[3]{x-\lambda}+\sqrt[3]{x-\lambda-1}=0$$
$$or\ \sqrt[3]{y+1}+\sqrt[3]{y}+\sqrt[3]{y-1}=0$$
$$\left(\sqrt[3]{y+1}+\sqrt[3]{y-1}\right)=-\sqrt[3]{y}\quad (1)$$

$$\left(\sqrt[3]{y+1}+\sqrt[3]{y-1}\right)^3=\left(-\sqrt[3]{y}\right)^3$$

$$y+1+y-1+3\sqrt[3]{y+1}.\sqrt[3]{y-1}\left(\sqrt[3]{y+1}+\sqrt[3]{y-1}\right)=-y$$

$$2y-3y^{\frac{1}{3}}\sqrt[3]{y+1}.\sqrt[3]{y-1}\overset{(1)}{=}-y$$
$$y=y^{\frac{1}{3}}\sqrt[3]{y+1}.\sqrt[3]{y-1}$$

$$y^3=y(y+1)(y-1)$$

$$y(y^2-y^2+1)=0\ or\ y=0$$

$$Solutions\ y=x-\lambda\ \ or\ x=\lambda\ (as\ y=0)$$