

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt{9+x} + \sqrt{9-x} + \sqrt{81-x^2} = 15$$

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$$\text{Denote: } \begin{cases} a = \sqrt{9+x} \\ b = \sqrt{9-x} \end{cases} \Rightarrow \begin{cases} a^2 + b^2 = 18 \\ a + b + ab = 15 \end{cases}$$

$$\text{Denote: } \begin{cases} S = a + b \\ P = ab \end{cases} \Rightarrow \begin{cases} S^2 - 2P = 18 \\ S + P = 15 \end{cases}$$

$$P = 15 - S \Rightarrow S^2 - 2(15 - S) = 18$$

$$S^2 - 30 + 2S - 18 = 0 \Rightarrow S^2 + 2S - 48 = 0, \quad \Delta = 4 + 192 = 196$$

$$S = \frac{-2 + 14}{2} = \frac{12}{2} = 6; P = 15 - 6 = 9$$

$$\Rightarrow \begin{cases} a + b = 6 \\ ab = 9 \end{cases} \Rightarrow a = b = 3 \Rightarrow \begin{cases} \sqrt{9+x} = 3 \\ \sqrt{9-x} = 3 \end{cases} \Rightarrow \begin{cases} 9+x = 9 \\ 9-x = 9 \end{cases} \Rightarrow x = 0$$