

ROMANIAN MATHEMATICAL MAGAZINE

Let be $\lambda > 0$ fixed. Solve for R the equation:

$$\left(\frac{x^2 + x}{x^2 + \lambda}\right)^3 - \left(\frac{x - \lambda}{x^2 + \lambda}\right)^3 = 1$$

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Solution by Shirvan Tahirov-Azerbaijan

$$a^3 - b^3 = (a - b)^3 - 3ab(a - b)$$

$$\left(\frac{x^2 + x}{x^2 + \lambda}\right)^3 - \left(\frac{x - \lambda}{x^2 + \lambda}\right)^3 = 1$$

$$\left(\frac{x^2 + x}{x^2 + \lambda} - \frac{x - \lambda}{x^2 + \lambda}\right)^3 + 3 \times \frac{x^2 + x}{x^2 + \lambda} \times \frac{x - \lambda}{x^2 + \lambda} \left(\frac{x^2 + x}{x^2 + \lambda} - \frac{x - \lambda}{x^2 + \lambda}\right) = 1$$

$$1 + \frac{3x(x + 1)(x - \lambda)}{(x^2 + \lambda)^2} = 1$$

$$3x(x + 1)(x - \lambda) = 0, \quad x_1 = 0, \quad x_2 = -1 \text{ and } x_3 = \lambda$$

Set solutions : $\{-1 ; 0 ; \lambda\}$