

ROMANIAN MATHEMATICAL MAGAZINE

Let $\lambda \geq 0$ fixed. Solve for reals:

$$\sqrt{\lambda + x} + \sqrt{\lambda - x} + \sqrt{\lambda^2 - x^2} = \lambda + 2\sqrt{\lambda}$$

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$$\begin{cases} \lambda + x \geq 0 \Rightarrow x \geq -\lambda \\ \lambda - x \geq 0 \Rightarrow x \leq \lambda \end{cases} \Rightarrow x \in [-\lambda; \lambda]$$

$$A = \sqrt{\lambda + x} + \sqrt{\lambda - x} \quad (A \geq 0)$$

$$A^2 = 2\lambda + 2\sqrt{\lambda^2 - x^2} \Rightarrow \sqrt{\lambda^2 - x^2} = \frac{A^2 - 2\lambda}{2}$$

$$A + \frac{A^2 - 2\lambda}{2} = \lambda + 2\sqrt{\lambda} \Rightarrow A^2 + 2A - (4\lambda + 4\sqrt{\lambda}) = 0$$

$$\Delta = 4 + 16\lambda + 16\sqrt{\lambda} = [2(1 + 2\sqrt{\lambda})]^2$$

$$A = \frac{-2 \pm 2(1 + 2\sqrt{\lambda})}{2} \xrightarrow{A \geq 0} A = 2\sqrt{\lambda}$$

$$\sqrt{\lambda^2 - x^2} = \frac{4\lambda - 2\lambda}{2} = \lambda \Rightarrow \lambda^2 - x^2 = \lambda^2 \Rightarrow x = 0$$