ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt{4+x}+\sqrt{4-x}+\sqrt{16-x^2}=8$$

Proposed by Marin Chirciu – Romania

Solution by Daniel Sitaru – Romania

$$\text{Denote: } \begin{cases} a=\sqrt{4+x} \\ b=\sqrt{4-x} \end{cases}; \begin{cases} S=a+b>0 \\ P=ab>0 \end{cases}$$

$$\Rightarrow \begin{cases} a+b+ab=8 \\ a^2+b^2=8 \end{cases} \Rightarrow \begin{cases} S+P=8 \\ S^2-2P=8 \end{cases} \Rightarrow P=8-s$$

$$S^2-2(8-S)=8 \Rightarrow S^2-16+2S-8=0$$

$$S^2+2S-24=0 \Rightarrow \Delta=100$$

$$S=\frac{-2+10}{2}=4 \Rightarrow P=8-4=4$$

$$\begin{cases} S=4 \\ P=4 \end{cases} \Rightarrow \begin{cases} a=2 \\ b=2 \end{cases} \Rightarrow \begin{cases} \sqrt{4+x}=2 \\ \sqrt{4-x}=2 \end{cases} \Rightarrow x=0$$