

ROMANIAN MATHEMATICAL MAGAZINE

Solve for reals:

$$3x^2 + 2\sqrt{x^2 + 3} = -4x + 3$$

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$$\begin{aligned} 3x^2 + 2\sqrt{x^2 + 3} = -4x + 3 &\iff (2x + 1)^2 = \left(\sqrt{x^2 + 3} - 1\right)^2 \\ \iff \begin{cases} 2x + 1 = \sqrt{x^2 + 3} - 1 \\ 2x + 1 = 1 - \sqrt{x^2 + 3} \end{cases} &\iff \begin{cases} \sqrt{x^2 + 3} = 2x + 2 & (x \geq -1) \\ \sqrt{x^2 + 3} = -2x & (x \leq 0) \end{cases} \\ \iff \begin{cases} x^2 + 3 = 4x^2 + 8x + 4 & (x \geq -1) \\ x^2 + 3 = 4x^2 & (x \leq 0) \end{cases} &\iff \begin{cases} 3x^2 + 8x + 1 = 0 & (x \geq -1) \\ 3x^2 = 3 & (x \leq 0) \end{cases} \\ &\iff \begin{cases} x = \frac{-4 + \sqrt{13}}{3} \\ x = -1 \end{cases} \end{aligned}$$