

ROMANIAN MATHEMATICAL MAGAZINE

Solve for reals:

$$\sqrt[20]{x} - \sqrt[30]{x} - 3\sqrt[60]{x} = 9$$

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Solution by Adgozel Adgozelov – Azerbaijan

For $x > 0$, let $t = x^{\frac{1}{60}} > 0$. Then:

$$\begin{aligned}x^{\frac{1}{20}} - x^{\frac{1}{30}} - 3x^{\frac{1}{60}} = 9 &\iff t^3 - t^2 - 3t - 9 = 0 \iff (t-3)(t^2 + 2t + 3) = 0 \\&\iff (t-3)((t+1)^2 + 2) = 0 \implies t-3 = 0 \quad [\\&\implies t = 3 \implies x^{\frac{1}{60}} = 3 \implies x = 3^{60}\end{aligned}$$

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