

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$16x^2 - 56x + 51 = \sqrt{4x - 6} + \sqrt{8 - 4x}$$

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Domain: $4x - 6 \geq 0$ or, $x \geq \frac{3}{2}$ and $8 - 4x \geq 0$ or, $x \leq 2$ so $\frac{3}{2} \leq x \leq 2$

We know that upward opening parabola $y = ax^2 + bx + c$

$$\text{min at } x = -\frac{b}{2a}, y_{\min.} = c - \frac{b^2}{4a}$$

$$\text{so } y = 16x^2 - 56x + 51 \text{ min. at } x = -\frac{-56}{2 \times 16} = \frac{7}{4} \text{ and } y_{\min} = 51 - \frac{56 \times 56}{4 \times 16} = 2$$

$$\begin{aligned} \text{then } 16x^2 - 56x + 51 &\geq 2 \text{ and } \sqrt{4x - 6} + \sqrt{8 - 4x} \stackrel{CBS}{\leq} \sqrt{2(4x - 6 + 8 - 4x)} \\ &= 2 \text{ in } \left[\frac{3}{2}, 2\right] \text{ so, L.H.S} \geq 2 \text{ and} \end{aligned}$$

$$\text{R.H.S} \leq 2 \text{ equality possible only when both equal to 2 or } x = \frac{7}{4}$$