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If $a > 1, \lambda \in R$ then solve for real numbers:

$$a^{\sqrt{x-\lambda-1}} = \frac{1}{\sqrt{x-\lambda}}$$

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$$Domain\ x-\lambda-1 \ge 0\ or\ x \ge \lambda+1\ let\ t = \sqrt{x-\lambda-1} \Rightarrow x-\lambda = t^2+1$$

$$a^{\sqrt{x-\lambda-1}} = \frac{1}{\sqrt{x-\lambda}}\quad or\quad a^t = (t^2+1)^{-\frac{1}{2}}$$

$$t\log a = -\frac{1}{2}\log(t^2+1)\ \ or\ \ \log(t^2+1) + 2t\log a = 0$$

$$Let\ f(t) = \log(t^2+1) + 2t\log a\ \ then\ f'(t) = \frac{2t}{t^2+1} + 2loga > 0\ as$$

$$a > 1\ and \ge 0.\ \ So\ f(t)\ is\ increasing.$$

$$The\ only\ one\ solution\ is\ t = 0\ as\ f(0) = 0,$$

$$x-\lambda = t^2+1or\ x = \lambda+1\ as\ t = 0$$