ROMANIAN MATHEMATICAL MAGAZINE

$$\textit{Solve for } x \in \left(0, \frac{\pi}{2}\right):$$

$$\frac{1}{\cos^2 x} + \frac{1}{\cos^2\left(\frac{\pi}{3} - x\right)} + \frac{1}{\cos^2\left(\frac{\pi}{3} + x\right)} = 18$$

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Solution by Tapas Das-India

$$\frac{1}{\cos^2 x} + \frac{1}{\cos^2\left(\frac{\pi}{3} - x\right)} + \frac{1}{\cos^2\left(\frac{\pi}{3} + x\right)} = 18$$

$$\sec^2 x + \sec^2\left(\frac{\pi}{3} - x\right) + \sec^2\left(\frac{\pi}{3} + x\right) = 18$$

$$3 + \tan^2 x + \tan^2\left(\frac{\pi}{3} - x\right) + \tan^2\left(\frac{\pi}{3} + x\right) = 18$$

$$3 + \tan^2 x + \left(\frac{\sqrt{3} - tanx}{1 + \sqrt{3}tanx}\right)^2 + \left(\frac{\sqrt{3} + tanx}{1 - \sqrt{3}tanx}\right)^2 = 18$$

$$t^2 + \left(\frac{\sqrt{3} - t}{1 + \sqrt{3}t}\right)^2 + \left(\frac{\sqrt{3} + t}{1 - \sqrt{3}t}\right)^2 \overset{t=tanx}{=} 15$$

$$t^2 + \left(\frac{6 + 20t^2 + 6t^4}{(1 - 3t^2)^2}\right) = 15 \textit{ or } \frac{6 + 21t^2 + 9t^6}{(1 - 3t^2)^2} = 15$$

$$6 + 21t^2 + 9t^6 = 15(1 - 3t^2)^2 \textit{ or } 3t^6 - 45t^4 + 37t^2 - 3 = 0$$

$$3x^3 - 45x^2 + 37x - 3 \overset{x=t^2}{=} 0 \textit{ or } (x - 1)(3x^2 - 42x + 3) = 0$$

$$t^2 = x = 1 \textit{ and } (3x^2 - 42x + 3) = 0 \textit{ or } x = 7 \pm 4\sqrt{3}$$

$$t^2 = 1 \textit{ or } t = 1 \textit{ or } tanx = 1 \textit{ or } x = \frac{\pi}{4} \textit{ and } t^2 = 7 \pm 4\sqrt{3} \textit{ or}, t = \sqrt{7 \pm 4\sqrt{3}} \textit{ or},$$

$$tanx = 2 \pm \sqrt{3} \textit{ when } tanx = 2 + \sqrt{3} \textit{ or } x = \frac{5\pi}{12}, tanx = 2 - \sqrt{3} \textit{ or } x = \frac{\pi}{12}$$

$$\textit{So solutions}: x = \frac{5\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}$$