

ROMANIAN MATHEMATICAL MAGAZINE

If $\lambda > 0$ fixed then solve for reals:

$$(x - 2y)^2 + 5\lambda^2 = 6x(\lambda - 2x) + 6y(2\lambda - y)$$

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$$(x - 2y)^2 + 5\lambda^2 = 6x(\lambda - 2x) + 6y(2\lambda - y)$$

Expanding we get:

$$13x^2 - (4y + 6\lambda)x + (10y^2 - 12\lambda y + 5\lambda^2) = 0$$

$$\text{or } x = \frac{4y + 6\lambda \pm \sqrt{(4y + 6\lambda)^2 - 52(10y^2 - 12\lambda y + 5\lambda^2)}}{26} \quad (1)$$

$$\begin{aligned} \text{The discriminant } D &= (4y + 6\lambda)^2 - 52(10y^2 - 12\lambda y + 5\lambda^2) = \\ &= -504y^2 + 672\lambda y - 224\lambda^2 = -56(3y - 2\lambda)^2 \text{ clearly } D < 0. \end{aligned}$$

So real solution exist for fixed λ only when $(3y - 2\lambda)^2 = 0$ or $y = \frac{2\lambda}{3}$

$$\text{From (1): } x = \frac{\frac{4 \cdot 2\lambda}{3} + 6\lambda}{26} = \frac{\lambda}{3}$$

$$\text{So required solution } x = \frac{\lambda}{3}, y = \frac{2\lambda}{3}.$$