ROMANIAN MATHEMATICAL MAGAZINE

$$If\ x \in R\ \ then\ solve\ the\ equation:$$

$$\lfloor x^2 - 2 \rfloor + 2\lfloor x \rfloor = \lfloor x \rfloor^2\ ,\qquad \lfloor * \rfloor -\ floor\ function$$

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$$We\ have\ \begin{cases} x \in R \\ k \in Z \end{cases} \Rightarrow\ \lfloor x + k \rfloor = \lfloor x \rfloor + k$$

$$RHS = \lfloor x^2 - 2 \rfloor = \lfloor x^2 \rfloor - 2$$

$$n \in Z \to\ \lfloor x \rfloor = n$$

$$n \le x < n + 1 \Rightarrow\ \ n^2 \le x^2 < (n+1)^2$$

$$n^2 \le x^2 < n^2 + 2n + 1$$

$$n^2 \le x^2 \le n^2 + 2n$$

$$\lfloor x^2 \rfloor = 2 - 2\lfloor x \rfloor + \lfloor x \rfloor^2 \Rightarrow\ \ \lfloor x^2 \rfloor = 2 - 2n + n^2$$

$$n^2 - 2n + 2 \le x^2 < n^2 - 2n + 3$$

$$n^2 \le n^2 - 2n + 2 \le n^2 + 2n$$

$$n^2 \le n^2 - 2n + 2 < n^2 + 2n \Rightarrow \begin{cases} n \le 1 \\ n \ge \dfrac{1}{2} \end{cases} \to\ \ n \in Z$$

$$n = 1 \Rightarrow\ \ \lfloor x \rfloor = 1\ ,\qquad \lfloor x^2 \rfloor = 1$$

$$\begin{cases} n^2 \le x^2 < n^2 + 2n + 1 \\ n^2 - 2n + 2 \le x^2 < n^2 - 2n + 3 \end{cases} \Rightarrow \begin{cases} 1 \le x < 2 \\ 1 \le x < \sqrt{2} \end{cases}\quad x \in [1; \sqrt{2})$$