ROMANIAN MATHEMATICAL MAGAZINE

Solve the equation

$$\lfloor x\rfloor\cdot x+\lfloor x\rfloor\cdot\{x\}+\{x\}\cdot x=2000$$

$\lfloor * \rfloor -$ *floor function*, $\{*\}-$ *fractional part function*

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Solution by Amin Hajiyev-Azerbaijan

$$x=\lfloor x\rfloor+\{x\}=n+m\ \begin{cases}\lfloor x\rfloor=n\\ \{x\}=m,\ \ 0\le m<1\end{cases}$$

$$\lfloor x\rfloor\cdot x+\lfloor x\rfloor\cdot\{x\}+\{x\}\cdot x=2000$$

$$\to\ \ n\cdot(n+m)+m\cdot n+m\cdot(n+m)=2000$$

$$(n+m)^2+m\cdot n=2000\to\ n^2+m^2+3mn=2000$$

$$0\le m<1\to\ m=0\ \ n^2=2000\ \ n=\sqrt{2000}=20\sqrt{5}$$

$$n\le\lfloor 20\sqrt{5}\rfloor=44$$

$$m=1\to\ n^2+3n-1999=0$$

$$n=\frac{-3+\sqrt{9+7996}}{2}=\frac{\sqrt{8005}}{2}-1,5\approx\frac{89,5-3}{2}\approx 43.25$$

$$n\ge 44\to\ \ 44\le n\le 44\ \ \ n=44$$

$$m^2+132m+1936-2000=0\to\ m^2+132m-64=0$$

$$m=\frac{-132+\sqrt{132^2+256}}{2}=\frac{\sqrt{17680}-132}{2}=\frac{4\sqrt{1105}}{2}-66=2\sqrt{1105}-66$$

$$x=\lfloor x\rfloor+\{x\}=n+m=44+2\sqrt{1105}-66=2\sqrt{1105}-22$$

$$x=2\sqrt{1105}-22$$