ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\cot^2(x) = \lfloor\cos^2(x)\rfloor + \{\sin^2(x)\}$$

$\lfloor * \rfloor$ − *floor function* , $\{*\}$ − *fractional part function*

Proposed by Mais Hasanov-Azerbaijan

Solution by Amin Hajiyev-Azerbaijan

$$\lfloor\cos^2(x)\rfloor \to 0 \le \cos^2(x) \le 1; \{\sin^2(x)\} \to 0 \le \sin^2(x) \le 1$$

$$\{\sin^2(x)\} = \sin^2(x) - \lfloor\sin^2(x)\rfloor, \qquad 0 \le \sin^2(x) < 1 \longrightarrow \lfloor\sin^2(x)\rfloor = 0$$

$$\{\sin^2(x)\} = \sin^2(x) - 0 = \sin^2(x)$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} \to \sin(x) \ne 0 \;\; x \ne \pi k \to \lfloor\cos^2(x)\rfloor = 0$$

$$\cot^2(x) = \lfloor\cos^2(x)\rfloor + \{\sin^2(x)\}, \cot^2(x) = \sin^2(x)$$

$$\cos^2(x) = \sin^4(x) \to \sin^4(x) + \sin^2(x) - 1 = 0$$

$$\sin^2(x) = t \to t^2 + t - 1 = 0, t_{1;2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$t \ne \frac{-1-\sqrt{5}}{2} < 0, \qquad t = \frac{-1+\sqrt{5}}{2} > 0$$

$$t = \frac{\sqrt{5}-1}{2} \quad \sin^2(x) = t \to \sin(x) = \pm\sqrt{\frac{\sqrt{5}-1}{2}}$$

$$x_1 = \pm\arcsin\sqrt{\frac{\sqrt{5}-1}{2}} + \pi k$$

$$\sin^2(x) = 1 \to \sin(x) = \pm 1 \;\; \cos(x) = 0$$

$$\frac{\cos^2(x)}{\sin^2(x)} = \lfloor\cos^2(x)\rfloor + \{\sin^2(x)\}, \qquad 0 = \lfloor 0 \rfloor + \{1\} = 0$$

$$x_1 = \pm\arcsin\sqrt{\frac{1}{\varphi}} + \pi k, \qquad x_2 = \frac{\pi}{2} + \pi k; \quad k \in Z$$