ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt{2x^2-5x-42}\,\sqrt[4]{(2x^2+x-3)}+1=\left(\left\lfloor sin\left(\frac{\pi x}{7}\right)\right\rfloor+\left\lfloor cos\left(\frac{2\pi x}{3}\right)\right\rfloor\right)^2$$

$$\lfloor * \rfloor \rightarrow\ floor\ function$$

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$$LHS=\sqrt{2x^2-5x-42}\,\sqrt[4]{(2x^2+x-3)}+1>0$$

$$RHS=\left(\left\lfloor sin\left(\frac{\pi x}{7}\right)\right\rfloor+\left\lfloor cos\left(\frac{2\pi x}{3}\right)\right\rfloor\right)^2\geq 0$$

$$\begin{cases}\sqrt{2x^2-5x-42}\geq 0\\ \sqrt[4]{(2x^2+x-3)}\geq 0\end{cases}\rightarrow\begin{cases}(2x+7)(x-6)\geq 0\\ (2x+3)(x-1)\geq 0\end{cases}\begin{matrix}x\geq 6,x\leq-\frac{7}{2}\\ x\geq 1\,,x\leq-\frac{3}{2}\end{matrix}$$

$$x\in\left(-\infty;\ -\frac{7}{2}\right]\cup[6;\ +\infty)\quad LHS=1\rightarrow\quad x_1=-\frac{7}{2}\ \ x_2=6$$

$$-1\leq sin(\alpha)\leq 1\quad -1\leq cos(\alpha)\leq 1$$

$$RHS=\left(\left\lfloor sin\left(\frac{\pi x}{7}\right)\right\rfloor+\left\lfloor cos\left(\frac{2\pi x}{3}\right)\right\rfloor\right)^2=\{0;1;4\}$$

$$RHS=LHS=1\rightarrow\quad RHS\neq\{0;4\}$$

$$x_1=-\frac{7}{2}\rightarrow\ RHS=\left(\left\lfloor sin\left(-\frac{\pi}{2}\right)\right\rfloor+\left\lfloor cos\left(-\frac{7\pi}{3}\right)\right\rfloor\right)^2=(-1+0)^2=1$$

$$x_2=6\rightarrow\ RHS=\left(\left\lfloor sin\left(\frac{6\pi}{7}\right)\right\rfloor+\lfloor cos(4\pi)\rfloor\right)^2=(0+1)^2=1$$

$$x\rightarrow\left(-\frac{7}{2};6\right)$$