ROMANIAN MATHEMATICAL MAGAZINE

$$\textit{If } x, y \in Z \textit{ solve}$$

$$(x+1)^3+(y-1)^3+1=(x+y+1)^2$$

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$$(x+1)^3+(y-1)^3+1=(x+y+1)^2$$

$$\begin{cases} x+1=a \\ y-1=b \end{cases} \to (x+1)^3+(y-1)^3+1=(x+1+y-1+1)^2$$

$$a^3+b^3+1=(a+b+1)^2$$

$$(a+b)(a^2-ab+b^2)+1=a^2+b^2+2ab+2a+2b+1$$

$$(a+b)(a^2-ab+b^2)=(a+b)^2+2(a+b)$$

$$(a+b)(a^2-ab+b^2-a-b-2)=0 \to a+b=0$$

$$2a^2-2ab+2b^2-2a-2b-4=0$$

$$a^2-2ab+b^2+a^2-2a+1+b^2-2b+1-6=0$$

$$(a-b)^2+(a-1)^2+(b-1)^2=6=1^2+1^2+2^2$$

$$a-b=\pm1, a-1=\pm1, b-1=\pm2 \to a,b\ (2;3)(0;-1)$$

$$a-b=\pm2, a-1=\pm1, b-1=\pm1 \to (2;0)(0;2)$$

$$a-b=\pm1, a-1=\pm2, b-1=\pm1 \to\ a,b\ (3;2)(-1;0)$$

$$a,b \to (2;3)(0;-1)(2;0)(0;2)(3;2)(-1;0)$$

$$\begin{cases} a=x+1 \\ b=y-1 \end{cases} \to\ x,y \in \{(2;3),(-2;1),(1;4),(-1;0),(1;1),(-1;3)\}$$

$$a+b=0 \to\ x+y=0\ , x=-y$$

$$\textit{General solution}: x=-y, \qquad \textit{where } y \in Z$$