ROMANIAN MATHEMATICAL MAGAZINE

Solve for natural numbers:

$$(x+2)^{\frac{1}{x}}+(x+1)^x=2^{2x}-5$$

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Solution by Tapas Das-India

$for\ x=1$

$$L.H.S=(1+2)^1+2^1=5, R.H.S=2^2-5=-1\ so\ L.H.S\neq R.H.S$$

$for\ x=2$

$$L.H.S=(2+2)^{\frac{1}{2}}+3^2=11, R.H.S=2^4-5=11\ so\ L.H.S=R.H.S$$

$$so\ x=2\ is\ a\ solution.$$

$for\ x\geq 3\ , (x+2)^{\frac{1}{x}}>1\ and\ (x+1)^x>4^x=2^{2x}$

$$then (x+2)^{\frac{1}{x}}+(x+1)^x>2^{2x}+1>2^{2x}-5$$

$$so\ for\ n\geq 3\ equality\ impossible.$$

$$The\ only\ solution\ x=2$$