ROMANIAN MATHEMATICAL MAGAZINE

Find $n \in N$ such that:

$$\frac{1}{\sqrt{1^3+2^3}}+\frac{1}{\sqrt{1^3+2^3+3^3}}+\cdots+\frac{1}{\sqrt{1^3+2^3+\cdots+n^3}}=\frac{2025}{2027}$$

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Solution by Tapas Das-India

$$1^3+2^3+3^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2 \ then \ \sqrt{1^3+2^3+3^3+\cdots+n^3}=\frac{n(n+1)}{2}$$

$$\frac{1}{\sqrt{1^3+2^3}}+\frac{1}{\sqrt{1^3+2^3+3^3}}+\cdots+\frac{1}{\sqrt{1^3+2^3+\cdots+n^3}}=\sum_{n=2}^{n}\frac{2}{n(n+1)}=$$

$$=2\sum_{n=2}^{n}\left(\frac{1}{n}-\frac{1}{n+1}\right)=2\left(\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right)=$$

$$=2\left(\frac{1}{2}-\frac{1}{n+1}\right)=\frac{n-1}{n+1}$$

$$\frac{n-1}{n+1}=\frac{2025}{2027} \ or \ n=2026$$