

ROMANIAN MATHEMATICAL MAGAZINE

Solve for reals :

$$x^2 + 2(2 + \sqrt{x-1}) = 5x$$

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$$x^2 + 2(2 + \sqrt{x-1}) = 5x, \quad x \geq 1$$

$$x^2 + 4 + 2\sqrt{x-1} - 5x = 0, \quad (x-1)(x-4) + 2\sqrt{x-1} = 0$$

$$\sqrt{x-1} = y, \quad x = y^2 + 1, \quad (y^2 + 1 - 1)(y^2 + 1 - 4) + 2y = 0$$

$$y^2(y^2 - 3) + 2y = 0, \quad y^4 - 3y^2 + 2y = 0, \quad y(y^3 - 3y + 2) = 0$$

$$y(y-1)^2(y+2) = 0$$

$$y = 0, \quad y = 1, \quad y = -2$$

$$\begin{cases} \sqrt{x-1} = 0, & x = 1 \\ \sqrt{x-1} = 1, & x = 2 \end{cases}$$