ROMANIAN MATHEMATICAL MAGAZINE

Solve for integers:

$$\left(\sqrt{x}-\sqrt{y}\right)^4 = 3361-\sqrt{11296320}$$

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$$\left(\sqrt{x}-\sqrt{y}\right)^4 = \left(x+y-2\sqrt{xy}\right)^2 = (x+y)^2+4xy-4\sqrt{xy}(x+y)$$

$$now, (x+y)^2+4xy-4\sqrt{xy}(x+y) = 3361-\sqrt{11296320}$$

$$comparing\ we\ get, (x+y)^2+4xy = 3361 \quad (1)$$

$$4\sqrt{xy}(x+y) = \sqrt{11296320} \quad (2)$$

$$Let\ x+y = s\ and\ xy = p\ then\ from (1)\&(2) we\ have$$

$$s^2+4p = 3361\ (3)$$

$$4s\sqrt{p} = \sqrt{11296320}\ or, s^2p = \frac{11296320}{16} = 706020\ (4)$$

$$From\ (3)\&\ (4) we\ have\ s^2p = 706020\ or\ s^2\left(\frac{3361-s^2}{4}\right) \overset{(3)}{=} 706020$$

$$s^2(3361-s^2) = 2824080\ or\ s^4-3361s^2+2824080 = 0$$

$$s^2 = \frac{3361\pm\sqrt{(3361)^2-4.1.2824080}}{2} = \frac{3361\pm 1}{2} = 1681, 1680$$

$$as\ we\ solve\ the\ equation\ for\ Z^+$$

$$for\ this\ we\ take\ s^2 = (x+y)^2 = 1681 = 41^2$$

$$s = x+y = 41\ and\ p = xy \overset{from\ (3)}{=} \frac{3361-s^2}{4} = \frac{3361-1681}{4} = 420$$

$$We\ have\ x+y = 41\ and\ xy = 420$$

$$To\ find\ (x,y)\ we\ construct\ an\ equation\ with\ roots\ x\ \&\ y,$$

$$equation\ will\ be\ t^2-(x+y)t+xy = 0\ or$$

$$t^2-41t+420 = 0\ or (t-21)(t-20) = 0\ \ or\ \ t = 21, 20$$

$$Required\ solution\ x = 21, y = 20\ or\ x = 20, y = 21.$$