ROMANIAN MATHEMATICAL MAGAZINE

Solve for integers:

$$\sqrt{x+y}+2=\sqrt{x}+\sqrt{y}$$

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$$Let\ a=\sqrt{x}, b=\sqrt{y}\ therefore\ \ x=a^2, y=b^2\ (as\ x,y\in Z^+, x,y>0)$$

$$\sqrt{x+y}+2=\sqrt{x}+\sqrt{y}\ or\ \sqrt{a^2+b^2}+2=a+b\ or\ \sqrt{a^2+b^2}=a+b-2$$

$$or\left(\sqrt{a^2+b^2}\right)^2=(a+b-2)^2\ \left(as\ \sqrt{a^2+b^2}>0\ then\ R.H.S\ part\ a+b-2>0\right)$$

$$a^2+b^2=a^2+b^2+4+2ab-4a-4b\ or\ 2(ab-2a-2b+2)=0$$

$$ab-2a-2b=-2\ or, ab-2a-2b+4=-2+4\ or, (a-2)(b-2)=2\ (1)$$

$$Integer\ factorization\ of\ 2\ are\ (1,2),(2,1),(-1,-2),(-2,-1)$$

$$From\ (1)\ if\ a-2=1, b-2=2\ then\ a=3, b=4$$

$$if\ a-2=2, b-2=1\ then\ a=4, b=3$$

$$if\ a-2=-1, b-2=-2\ then\ a=1, b=0$$

$$if\ a-2=-2, b-2=-1\ then\ a=0, b=1$$

$$Now, a=3, b=4\Rightarrow\ x=9, y=16\ (as\ \ x=a^2, y=b^2\ )$$

$$a=4, b=3\Rightarrow\ x=16, y=19,$$

$$a=1, b=0\Rightarrow\ x=1, y=0\ does\ not\ satisfy\ given\ equation$$

$$a=0, b=1\Rightarrow\ x=0, y=1, does\ not\ satisfy\ given\ equation$$

$$so\ the\ required\ solution\ x=9, y=16\ or, x=16, y=9$$