ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt[3]{2034-x}-\sqrt{x-2025}=x^2-2025x-2025$$

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Solution by Tapas Das-India

$$Clearly\ x-2025\geq 0\ or\ x\geq 2025$$

$$Let\ \ g(x)=(x^2-2025x-2025)-\left(\sqrt[3]{2034-x}-\sqrt{x-2025}\right)$$
$$g'(x)=2x-2025+\frac{1}{3}(2024-x)^{-\frac{2}{3}}+\frac{1}{2}(x-2025)^{\frac{1}{2}}$$
$$g'(x)>0\ for\ x>2025\Rightarrow g(x)\ is\ strictly\ increasing\ ,therefore\ g(x)=0$$
$$has\ at\ most\ one\ solution\ .$$

$$Clearly\ \ g(2026)=2026(2026-2025)-2025-(2-1)=0$$
$$so\ x=2026\ is\ the\ only\ solution.$$