ROMANIAN MATHEMATICAL MAGAZINE

If $\lambda \in \mathbb{N}, \lambda \geq 2$ then solve for naturals:

$$\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+3+\cdots+x}=\frac{2\lambda}{2\lambda+1}$$

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$$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+3+\cdots+x}=\frac{4\lambda+1}{2\lambda+1}$$

$$\sum_{n=1}^{x}\frac{1}{\sum_{k=1}^{n}k}=\frac{4\lambda+1}{2\lambda+1}$$

$$Note:\rightarrow\begin{cases}1+2+3+\cdots+x=S\\ x+(x-1)+(x-2)+\cdots+1=S\end{cases}$$

$$2S=\underbrace{(x+1)+(x+1)+(x+1)+\cdots+(x+1)}_{x}\rightarrow S=\frac{x(x+1)}{2}$$

$$\sum_{k=1}^{n}k=\frac{n(n+1)}{2}\rightarrow 2\sum_{n=1}^{x}\frac{1}{n(n+1)}=\frac{4\lambda+1}{2\lambda+1}$$

$$2\underbrace{\left(\sum_{n=1}^{x}\frac{1}{n}-\sum_{n=1}^{x}\frac{1}{n+1}\right)}_{Teleskopik\ sum}=\frac{4\lambda+1}{2\lambda+1}\rightarrow 2\left(1-\frac{1}{x+1}\right)=\frac{4\lambda+1}{2\lambda+1}$$

$$\frac{x-1}{x+1}=\frac{2\lambda}{2\lambda+1}\rightarrow\frac{x+1}{x-1}=1+\frac{1}{2\lambda}\rightarrow\frac{2}{x-1}=\frac{1}{2\lambda}$$

$$x=4\lambda+1$$