ROMANIAN MATHEMATICAL MAGAZINE

If $\lambda > 2$ is a fixed even integer, solve for integers:

$$\lambda^2 x^2(y-1) - 2\lambda x - y - 1 = 0$$

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$$\lambda^2 x^2(y-1) - 2\lambda x - (y-1) - 2 = 0$$

$$(y-1)(\lambda^2 x^2 - 1) - 2(\lambda x + 1) = 0$$

$$(y-1)(\lambda x - 1)(\lambda x + 1) - 2(\lambda x + 1) = 0$$

$$(\lambda x + 1)[(y-1)(\lambda x - 1) - 2] = 0$$

Case 1: $\lambda x + 1 = 0 \Rightarrow x = -\frac{1}{\lambda} \notin \mathbb{Z}$ (since $\lambda > 2$)

Case 2: $(y-1)(\lambda x - 1) = 2 \Rightarrow (\lambda x - 1) \in \{-2, -1, 1, 2\}$

- $\lambda x - 1 = 1 \Rightarrow x = \frac{2}{\lambda} \notin \mathbb{Z}$ (since $\lambda > 2$)
- $\lambda x - 1 = 2 \Rightarrow x = \frac{3}{\lambda} \notin \mathbb{Z}$ (since λ is even)
- $\lambda x - 1 = -2 \Rightarrow x = -\frac{1}{\lambda} \notin \mathbb{Z}$
- $\lambda x - 1 = -1 \Rightarrow x = 0 \Rightarrow y - 1 = -2 \Rightarrow y = -1$

Unique integer solution: $(x, y) = (0, -1)$