

ROMANIAN MATHEMATICAL MAGAZINE

Solve for naturals:

$$4(x + y + z) + xyz = 2(xy + yz + zx) + 9$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

We know that:

$$(x - 2)(y - 2)(z - 2) = xyz - 2(xy + yz + zx) + 4(x + y + z) - 8$$

$$\Rightarrow 4(x + y + z) + xyz - 2(xy + yz + zx) = (x - 2)(y - 2)(z - 2) + 8 \quad (1)$$

from the given condition, $4(x + y + z) + xyz = 2(xy + yz + zx) + 9$

$$\Rightarrow 4(x + y + z) + xyz - 2(xy + yz + zx) = 9$$

$$\Rightarrow (x - 2)(y - 2)(z - 2) + 8 \stackrel{(1)}{=} 9 \Rightarrow (x - 2)(y - 2)(z - 2) = 1$$

Thus we need integer triplets whose product is 1

Case – 1

1.1.1 = 1, for this $x - 2 = y - 2 = z - 2 = 1$ or, $x = y = z = 3$

solution: $(x, y, z) = (3, 3, 3)$

Case – 2

$$(-1) \cdot (-1) \cdot (1) = 1 \text{ (and permutation)}$$

for this $x - 2 = -1, y - 2 = -1, z - 2 = 1$ which gives $x = 1, y = 1, z = 3$

solutions: $(x, y, z) = (1, 1, 3), (1, 3, 1), (3, 1, 1)$