ROMANIAN MATHEMATICAL MAGAZINE

Solve for integers:

$$x^4+6x^3+13x^2-4y^2+12x+8y+12=0$$

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$$x^4+6x^3+13x^2-4y^2+12x+8y+12=0$$
$$or,(x^2+3x+2)^2-4(y-1)^2=-12$$
$$or,4(y-1)^2-(x^2+3x+2)^2=12$$
$$or,4(y-1)^2-\left((x+1)(x+2)\right)^2=12\ (1)$$

$$clearly\ (x+1),(x+2)\ are\ two\ consecutive\ numbers$$
$$so\ (x+1)(x+2) is\ multiple\ of\ 2\ , let\ (x+1)(x+2)=2m\ , m\in Z$$
$$from\ (1) we\ get\ 4(y-1)^2-4m^2=12\ \ or,(y-1)^2-m^2=3$$
$$or,(y-1+m)(y-1-m)=3$$
$$possible\ pairs\ are:$$
$$(y-1+m)(y-1-m)=(3,1),(-1,-3),(-3,-1),(1,3)$$

$$y-1+m=3, y-1-m=1\Rightarrow y-1=2, m=1\ or, y=3, m=1$$
$$m=1\Rightarrow(x+1)(x+2)=2m\ or, x^2+3x+2=2\ or, x=-3,0$$
$$solution=(x,y)=(-3,3),(0,3)$$

$$y-1+m=-1, y-1-m=-3\Rightarrow y-1=-2, m=1\ or, y=-1, m=1$$
$$m=1\Rightarrow(x+1)(x+2)=2m\ or, x^2+3x+2=2\ or, x=-3,0$$
$$solution=(x,y)=(-3,-1),(0,-1)$$

$$y-1+m=-3, y-1-m=-1\Rightarrow y-1=-2, m=-1\ or, y=3, m=-1$$
$$m=-1\Rightarrow(x+1)(x+2)=2m\ or, x^2+3x+2=-2\ or, x^2+3x+4=0$$
$$Discriminant=9-16<0\ no\ solution\ and\ similar\ for\ the\ pair\ (3,1)$$
$$solution=(x,y)=(-3,3),(0,3),(-3,-1),(0,-1)$$