ROMANIAN MATHEMATICAL MAGAZINE

Solve for integers:

$$(x+1)(x+2)(x+3)(x+4)=y^2-12$$

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Solution by Tapas Das-India

$$(x+1)(x+4)=x^2+5x+4$$
$$(x+2)(x+3)=x^2+5x+6\,,$$
$$Let\ u=x^2+5x+5\ \ then\ (x+1)(x+4)=u-1\ \&\ (x+2)(x+3)=u+1$$
$$(x+1)(x+2)(x+3)(x+4)=y^2-12$$
$$or,(u+1)(u-1)=y^2-12$$
$$or,u^2-1=y^2-12\ or,y^2-u^2=11$$

$$11\ is\ a\ prime\ number\ ,so\ possible\ pairs\ are:$$

$$(y-u,y+u)=(1,11),(-1,-11),(11,1),(-11,-1)$$
$$y-u=1,y+u=11\Rightarrow y=6,u=5$$
$$y-u=-1,y+u=-11\Rightarrow y=-6,u=5$$
$$y-u=11,y+u=1\Rightarrow y=6,u=-5$$
$$y-u=-11,y+u=-1\Rightarrow y=-6,u=5$$

$for\ u=5\ \&\ y=6\ ,y=-6\ we\ get\ x^2+5x+5=5\ or,x=0,-5$

$$solution\ (x,y)=(0,6),(0,-6),(-5,6),(-5,-6)$$

$for\ u=-5\ \&\ y=6\ we\ get\ x^2+5x+5=-5\ or,x^2+5x+10=0$

$$Discriminant=-15\ no\ integer\ solution\ exist$$

$for\ u=5\ \&\ y=6\ we\ get\ x^2+5x+5=5\ or,x=0,-5$