

ROMANIAN MATHEMATICAL MAGAZINE

Determine all pairs (x, y) of integers such that:

$$3x^2 + 10x + 5 = 9 \cdot 2^y$$

Proposed by Bibek Yadav-Nepal

Solution by Jenish Rijal-Nepal

It is clear that $y \geq 0$ and x is odd if $y > 0$.

Case ①: $y = 0, 1$.

$$3x^2 + 10x + 5 = \begin{cases} 9 \text{ for } y = 0 \Rightarrow x = \frac{-5 \pm \sqrt{37}}{3} \Rightarrow \text{No integer solution.} \\ 18 \text{ for } y = 1 \Rightarrow x = 1 \text{ and } \frac{-13}{3} \Rightarrow (x, y) = (1, 1). \end{cases}$$

Case ②: $y \geq 2$.

We know that x is odd $\Rightarrow x \equiv 1, 3 \pmod{4} \Rightarrow 10x \equiv 2 \pmod{4} \Rightarrow 3x^2 \equiv 3 \pmod{4}$

$\Rightarrow 3x^2 + 10x + 5 \equiv 3 + 2 + 5 \equiv 2 \pmod{4}$ but $RHS = 9 \cdot 2^y \equiv 0 \pmod{4} \because y \geq 2$.

\Rightarrow A contradiction! \Rightarrow No integer solution for $y \geq 2$.

\therefore The only possible solution is $(x, y) = (1, 1)$.