ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b>0\ \&\ a+b=4\sqrt{\frac{ab}{3}}\ then\ find$$

$$A=\left|\frac{a-b}{a+b}+\frac{a^2-b^2}{a^2+b^2}+\frac{a^3-b^3}{a^3+b^3}\right|$$

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Solution by Tapas Das-India

$$a+b=4\sqrt{\frac{ab}{3}}\ or,(a+b)^2=16\frac{ab}{3}\ or,3a^2-10ab+3b^2=0$$

$$(3a-b)(b-3a)=0\ or\ (a=3b\ \ or\ b=3a).If\ we\ take\ a=3b\ then:$$

$$\frac{a-b}{a+b}=\frac{3b-b}{3b+b}=\frac{1}{2},\frac{a^2-b^2}{a^2+b^2}=\frac{9b^2-b^2}{9b^2+b^2}=\frac{4}{5},\frac{a^3-b^3}{a^3+b^3}=\frac{27b^3-b^3}{27b^3+b^3}=\frac{13}{14}$$

$$\left|\frac{a-b}{a+b}+\frac{a^2-b^2}{a^2+b^2}+\frac{a^3-b^3}{a^3+b^3}\right|=\left|\frac{1}{2}+\frac{4}{5}+\frac{13}{14}\right|=\left|\frac{78}{85}\right|=\frac{78}{85}$$

$$similarly\ for\ b=3a\ ,A=\left|-\frac{78}{85}\right|=\frac{78}{85}$$