ROMANIAN MATHEMATICAL MAGAZINE

If $A,B\in M_n(\mathbb{R})$; $AB+9A+4B=O_n$ then:

$$det^2(A+4I_n)+det^2(B+9I_n)\geq 72$$

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Solution by Daniel Sitaru – Romania

$$(A+4I_n)(B+9I_n)=AB+9A+4B+36I_n=36I_n$$

$$\det\big((A+4I_n)(B+9I_n)\big)=\det(36I_n)$$

$$\det(A+4I_n)\cdot\det(B+9I_n)=36^2$$

$$|\det(A+4I_n)|\cdot|\det(B+9I_n)|=36^2$$

$$det^2(A+4I_n)+det^2(B+9I_n)\overset{AM-GM}{\geq}2\sqrt{|\det(A+4I_n)\cdot\det(B+9I_n)|}=$$

$$=2\sqrt{|\det(A+4I_n)|\cdot|\det(B+9I_n)|}=2\sqrt{36^2}=72$$

Equality holds for: $|\det(A+4I_n)|=|\det(B+9I_n)|$