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If $A, B \in M_2(\mathbb{R})$; $A^2B = \begin{pmatrix} -1 & 2 \\ 3 & 5 \end{pmatrix}$; $B^2A = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}$ then find:

$$M = \det A + \det B.$$

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$$A^2B = \begin{pmatrix} -1 & 2 \\ 3 & 5 \end{pmatrix} \Rightarrow \det(A^2B) = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} \Rightarrow$$

$$(\det A)^2 \cdot \det B = -5 + 6 \Rightarrow \det B = \frac{1}{(\det A)^2}$$

$$B^2A = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix} \Rightarrow \det(B^2A) = \begin{vmatrix} 1 & 2 \\ 2 & 6 \end{vmatrix} \Rightarrow (\det B)^2 \cdot \det A = 6 \cdot 1 - 2 \cdot 2 = 6 - 4 = 2$$

$$\frac{1}{(\det A)^4} \cdot \det A = 2 \Rightarrow \frac{1}{(\det A)^3} = 2 \Rightarrow \det A = \frac{1}{\sqrt[3]{2}}$$

$$\det B = \frac{1}{(\det A)^2} = \sqrt[3]{4}$$

$$M = \det A + \det B = \frac{1}{\sqrt[3]{2}} + \sqrt[3]{4} = \frac{3}{\sqrt[3]{2}}$$