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If $A, B \in M_2(\mathbb{R}); A^{2027} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}; B^{2025} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ then find:

$$M = \det A + \det B$$

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$$A^{2027} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \Rightarrow \det(A^{2027}) = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix}$$

$$(\det A)^{2027} = 2 \cdot 3 - 1 \cdot 5 \Rightarrow (\det A)^{2027} = 1 \Rightarrow \det A = 1$$

$$B^{2025} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \Rightarrow \det\left(B^{2025}\right) = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \Rightarrow$$

$$\Rightarrow (\det B)^{2025} = 1 \cdot 3 - 2 \cdot 2 \Rightarrow (\det B)^{2025} = -1 \Rightarrow \det B = -1$$

$$M = \det A + \det B = 1 - 1 = 0$$