ROMANIAN MATHEMATICAL MAGAZINE

Let $y>0$ fixed. The equation $\sqrt{x}+\sqrt{y-x}=\sqrt{y+yx-x^2}$ has 4 distinct real solutions x_1,x_2,x_3,x_4. Calculate $S=x_1+x_2+x_3+x_4$.

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$$\begin{cases}\sqrt{x}=a\\ \sqrt{y-x}=b\end{cases}\Rightarrow\begin{cases}x=a^2\\ y-x=b^2\end{cases}\Rightarrow a^2+b^2=y$$

$$\sqrt{x}+\sqrt{y-x}=\sqrt{y+yx-x^2}\Rightarrow a+b=\sqrt{y+(ab)^2}$$

$$(a+b)^2=y+(ab)^2$$

$$a^2+b^2+2ab=y+(ab)^2$$

$$a^2+b^2+2ab=a^2+b^2+(ab)^2$$

$$2ab=(ab)^2\Rightarrow\begin{cases}ab=0\\ ab=2\end{cases}\Rightarrow\begin{cases}\sqrt{x(y-x)}=0\\ \sqrt{x(y-x)}=2\end{cases}\Rightarrow$$

$$\begin{cases}\sqrt{x(y-x)}=0\\ \sqrt{x(y-x)}=2\end{cases}\Rightarrow\begin{cases}x^2-yx=0\\ x^2-yx+4=0\end{cases}\Rightarrow\begin{cases}x_1+x_2=y\\ x_3+x_4=y\end{cases}$$

$$S=x_1+x_2+x_3+x_4=2y$$