ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \lim_{x\to\infty}\left[\int_x^{x+\frac{3}{x}} t^2 \arcsin\left(\frac{1}{t}\right) dt\right]$$

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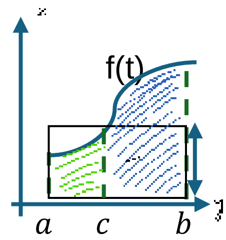
Mean Value Theorem for Integrals:

If f is continuous on $[a;\ b]$, then there exists $c \in [a;\ b]$ such that:

$$\int_a^b f(t)dt = f(c)(b-a)$$



$h = f(c)$

$f(t) = t^2 \sin^{-1}\left(\frac{1}{t}\right) \quad a = x, b = x + \frac{3}{x}$

$$\Omega = \lim_{x\to\infty}\left[\left(x+\frac{3}{x}-x\right)f(c)\right]$$
$$= 3\lim_{x\to\infty}\left[\frac{1}{x}\left(c^2 \sin^{-1}\left(\frac{1}{c}\right)\right)\right]$$

$$Asymptotik\ analysis:\quad x \le c \le x + \frac{3}{x} \to\ x\to\infty, \qquad x \approx c$$

Taylor series for: $\arcsin(y) = \sum_{n=0}^{\infty}\frac{(2n)!y^{2n+1}}{4^n(2n+1)(n!)^2} \sim y + O(y^3)\ as\ |y| \le 1$

$$x\to\infty, x\approx c \to \left|\frac{1}{c}\right| < 1 \quad \arcsin\left(\frac{1}{c}\right) \sim \frac{1}{c} + O\left(\frac{1}{c^3}\right)$$

$$\text{Therfore}\ \ \Omega = 3\lim_{x\to\infty}\left[\frac{1}{x}\cdot c^2\cdot\left(\frac{1}{c}+O\left(\frac{1}{c^3}\right)\right)\right] = 3\lim_{x\to\infty}\frac{c}{x} = 3$$