ROMANIAN MATHEMATICAL MAGAZINE

If $0 < a \le b$ then:

$$\int_a^b \int_a^b \frac{dxdy}{\sqrt{(1+x)(1+y)}} \le (b-a)\ln\frac{b+1}{a+1}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\frac{1}{\sqrt{(1+x)(1+y)}} = \sqrt{\frac{1}{1+x}\cdot\frac{1}{1+y}} \overset{AM-GM}{\le} \frac{1}{2}\left(\frac{1}{1+x}+\frac{1}{1+y}\right)$$

$$\int_a^b \int_a^b \frac{dxdy}{\sqrt{(1+x)(1+y)}} \le \frac{1}{2}\int_a^b \int_a^b \left(\frac{1}{1+x}+\frac{1}{1+y}\right) dxdy =$$

$$= \frac{1}{2}\int_a^b \int_a^b \frac{1}{1+x} dxdy + \frac{1}{2}\int_a^b \int_a^b \frac{1}{1+y} dxdy =$$

$$= \frac{1}{2}(y)_a^b.(\ln(1+x))_a^b + \frac{1}{2}(x)_a^b.(\ln(1+y))_a^b = (b-a)\ln\frac{b+1}{a+1}$$

Equality holds for a=b.