

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$I = \int_0^{\frac{\pi}{6}} \frac{\tan(x)}{(1 + \tan(x))^2} dx$$

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Solution by Amin Hajiyevev-Azerbaijan

$$\begin{aligned} I &= \int_0^{\frac{\pi}{6}} \frac{\tan(x)}{(1 + \tan(x))^2} dx \rightarrow \tan(x) = t, \frac{dt}{dx} = 1 + t^2 \\ I &= \int_0^{\frac{1}{\sqrt{3}}} \frac{t}{(1 + t)^2(1 + t^2)} dt = \frac{1}{2} \int_0^{\frac{1}{\sqrt{3}}} \left(\frac{1}{1 + t^2} - \frac{1}{(1 + t)^2} \right) dt = \\ &= \frac{1}{2} \left[\arctan(t) + \frac{1}{1 + t} \right]_0^{\frac{1}{\sqrt{3}}} = \frac{1}{2} \left(\frac{\pi}{6} + \frac{1}{1 + \frac{1}{\sqrt{3}}} - 1 \right) = \\ &= \frac{1}{2} \left(\frac{\pi}{6} - \frac{1}{1 + \sqrt{3}} \right) = \frac{\pi}{12} - \frac{\sqrt{3} - 1}{4} \end{aligned}$$