ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{6}} \frac{1+\tan^2(x)}{1+\sin(x)}dx$$

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$$\int_0^{\frac{\pi}{6}} \frac{1+\tan^2(x)}{1+\sin(x)}dx \overset{\tan(x)\to t}{\cong} \int_0^{\frac{\sqrt{3}}{3}} \frac{1}{1+\frac{t}{\sqrt{1+t^2}}}dt =$$

$$= \int_0^{\frac{\sqrt{3}}{3}} \frac{\sqrt{1+t^2}}{t+\sqrt{1+t^2}}dt = \int_0^{\frac{\sqrt{3}}{3}} \sqrt{1+t^2}\left(\sqrt{1+t^2}-t\right)dt =$$

$$= \int_0^{\frac{\sqrt{3}}{3}} \sqrt{1+t^2}dt - \int_0^{\frac{\sqrt{3}}{3}} t\sqrt{1+t^2}dt =$$

$$= \left(t+\frac{t^3}{3}\right)\Bigg|_0^{\frac{\sqrt{3}}{3}} - \frac{1}{2}\int_0^{\frac{\sqrt{3}}{3}} (1+t^2)^{\frac{1}{2}}d(1+t^2) =$$

$$\left(\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{27}\right) - \frac{1}{2}\cdot\frac{2}{3}(1+t^2)\left(\sqrt{1+t^2}\right)\Bigg|_0^{\frac{\sqrt{3}}{3}} = \frac{10\sqrt{3}}{27} - \frac{8\sqrt{3}}{27} + \frac{1}{3} = \frac{2\sqrt{3}}{27} + \frac{1}{3}$$