ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\int_0^{\frac{\pi}{6}}\frac{\tan(x)}{2+tan^2(x)}dx$$

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$$\int_0^{\frac{\pi}{6}}\frac{\tan(x)}{2+tan^2(x)}dx=\int_0^{\frac{\sqrt{3}}{3}}\frac{t}{2+t^2}\cdot\frac{dt}{1+t^2}=\frac{1}{2}\int_0^{\frac{\sqrt{3}}{3}}\frac{dt^2}{(2+t^2)(1+t^2)}=$$

$$Let\ \tan(x)=t\ \rightarrow\ dt=\frac{dx}{cos^2(x)}\rightarrow\ dx=\frac{dt}{1+t^2}$$

$$=\frac{1}{2}\int_0^{\frac{\sqrt{3}}{3}}\left(\frac{1}{1+t^2}-\frac{1}{2+t^2}\right)dt^2=$$

$$=\frac{1}{2}(\ln(1+t^2)-\ln(2+t^2))\Big|_0^{\frac{\sqrt{3}}{3}}=\frac{1}{2}\ln\left(\frac{1+t^2}{2+t^2}\right)\Bigg|_0^{\frac{\sqrt{3}}{3}}=\frac{1}{2}\left(\ln\left(\frac{4}{7}\right)-\ln\left(\frac{1}{2}\right)\right)=\frac{1}{2}\ln\left(\frac{8}{7}\right)$$

$$So:$$

$$\Omega=\int_0^{\frac{\pi}{6}}\frac{\tan(x)}{2+tan^2(x)}dx=\frac{1}{2}\ln\left(\frac{8}{7}\right)$$